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TWIST VELOCITY INFLUENCE ON LENGTHWISE FRACTURE OF INHOMOGENEOUS BARS UNDER TORSIONAL LOADING

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Abstract. *This paper is concerned with studying the influence of the twist velocity on lengthwise fracture of inhomogeneous load-carrying bar subjected to torsional loading. The bar under consideration has non-linear elastic behavior. The cross-section of the bar is a circle. The bar has three portions with different radius of the cross-section. The bar is under angles of twist that are time-dependent. The material of the bar is continuously inhomogeneous in radial direction. The influence of the twist velocity is taken into account by applying a non-linear stress-strain constitutive law that includes a term with the first derivative of the shear strain with respect to time. This constitutive law is used to develop a theoretical analysis of lengthwise fracture in terms of the strain energy release rate (SERR) with considering the twist velocity. Actually, obtaining of the SERR with taking into account the twist velocity is the basic aim of this paper. The parameters of the stressed and strained state of the twisted bar that are needed for deriving the SERR are obtained by analyzing the equilibrium of the bar portions. The energy balance in the bar is investigated to verify the SERR. Numerical results are obtained and reported in form of graphs for clarifying the effect of various factors and parameters on the SERR in continuously inhomogeneous bars under time-dependent twist.*

Keywords: *Inhomogeneous Bar, Twist Velocity, Lengthwise Fracture, Relaxation, Torsional Loading*

1. INTRODUCTION

The growing use of smoothly inhomogeneous materials in a variety of engineering applications necessitates carrying-out investigations of different aspects of their behavior [1]-[13]. Functionally graded sandwich beams and their dynamical behavior are analyzed numerically in [11]. A new numerical model for analyzing the vibration behavior of bi-directionally functionally graded beams is explored in [12]. Static and vibration analysis of functionally graded shell panels is carried-out in [13].

It is exceptionally important to investigate fracture in load-carrying applications of inhomogeneous materials since fracture brings about the dramatic reduction of load-resisting ability, rigidity, life span and safety of the structures, machines and devices [10], [14]-[16]. At the same time, there is a broad spectrum of factors and parameters with a high potential for affecting the fracture behavior of components of structures and mechanisms made of smoothly inhomogeneous engineering materials.

The diversity that is inherent to the fracture behavior of these materials imposes developing of different analyses that deal with specific aspects of the fracture problem like the crack geometry, type of loading, mechanical behavior of the material, shape of the structural component, boundary conditions, etc.

On this background the problem of lengthwise fracture is studied here. In particular, the aim of the current research is to determine how the SERR for a lengthwise cylindrical crack in a non-linear elastic bar under torsional loading behaves when the twist velocity is considered in the solution. It should be mentioned here that the difference of the current study from the lengthwise fracture studies of non-linear elastic inhomogeneous bars under torsion published in the literature [14] is that the current study accounts for the twist velocity (in fact, taking into account the twist velocity when determining the SERR is the basic aim of this study). The constitutive law that is applied here expresses the torsion induced shear stress as a non-linear function of the shear strain and its first derivative with respect to the time. The solution of the SERR that is determined by a method analyzing the balance of the energy accounts for the twist velocity. A method using the complementary strain energy is applied for verifying the solution. Numerical results are obtained by this solution. The results are reported in form of graphs showing how the SERR changes under influence of parameters of time-dependent twist and material inhomogeneity.

2. THEORETICAL STUDY

The analysis reported in this paper deals with the load-carrying bar shown in Fig. 1. The bar is under angles of twist, φ_{D1}, φ_{D2} and φ_{D4}, in sections, D_1, D_2 and D_4. Equations (1), (2) and (3) describe variation of φ_{D1}, φ_{D2} and φ_{D4} with time, t.

$$\varphi_{D1} = \upsilon_{D1} t \quad (1)$$

$$\varphi_{D2} = \upsilon_{D2} t \quad (2)$$

$$\varphi_{D4} = \upsilon_{D4} t \quad (3)$$

where υ_{D1}, υ_{D2} and υ_{D4} are parameters.

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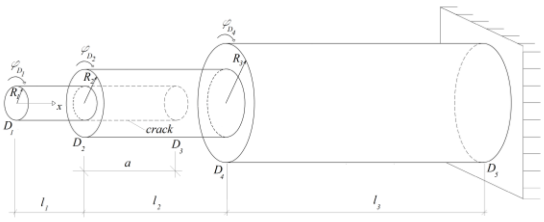

Figure 1. Bar with a lengthwise crack.

Equation (4) represents non-linear constitutive law that is applied in this paper to model the bar mechanical behavior under torsion with considering the influence of twist velocity [17].

$$\tau = \frac{\gamma}{H + L\gamma}\left(1 + P\frac{\dot{\gamma}}{\dot{\gamma}_0}\right) \tag{4}$$

where τ is the shear strain induced by the torsion, γ is the shear strain, $\dot{\gamma}$ is shear strain velocity, H, L, P and $\dot{\gamma}_0$ are material parameters. These material parameters change smoothly along the radius of the bar cross-section since the material is continuously inhomogeneous. The next four equations are used for describing the change of H, L, P and $\dot{\gamma}_0$.

$$H = H_{ct} e^{b\frac{R}{R_i}} \tag{5}$$

$$L = L_{ct} e^{f\frac{R}{R_i}} \tag{6}$$

$$P = P_{ct} e^{g\frac{R}{R_i}} \tag{7}$$

$$\dot{\gamma}_0 = \dot{\gamma}_{0ct} e^{h\frac{R}{R_i}} \tag{8}$$

where

$$0 \le R \le R_i \tag{9}$$

$$i = 1,2,3 \tag{10}$$

The following designations are used in Equations (5) – (9): H_{ct}, L_{ct}, P_{ct} and $\dot{\gamma}_{0ct}$ are the values of H, L, P and $\dot{\gamma}_0$ in the centre of the bar, b, f, g and h are parameters, R_i is the radius of the bar cross-sections in portions, D_1D_2, D_2D_4 and D_4D_5, as can be seen in Figure 1.

Equation (11) that is used for determining of the SERR, G, for the circular lengthwise crack in portion, D_2D_3, of the bar in Figure 1 is obtained by considering the balance of the energy.

$$G = \frac{1}{2\pi R_2}\left(T_{D1}\frac{\partial \varphi_{D1}}{\partial a} + T_{D2}\frac{\partial \varphi_{D2}}{\partial a} + T_{D4}\frac{\partial \varphi_{D4}}{\partial a} - \frac{\partial U}{\partial a} \right) \tag{11}$$

where T_{D1}, T_{D2} and T_{D4} are the external torsion moments in sections, D_1, D_2 and D_4, respectively, a is the crack length, U is the strain energy.

Equation (12) expresses U via the specific strain energies, u_{0D1D3}, u_{0D2D3}, u_{0D3D4} and u_{0D4D5}, in the bar portions.

$$U = \iiint_{(V_{D1D2})} u_{0D1D3}dV + \iiint_{(V_{D2D3})} u_{0D2D3}dV + \iiint_{(V_{D3D4})} u_{0D3D4}dV + \iiint_{(V_{D4D5})} u_{0D4D5}dV \tag{12}$$

where

$$u_{0D1D3} = \int \tau_{D1D3} d\gamma \tag{13}$$

Here, τ_{D1D3} is the shear stress in portion, D_1D_3. The specific strain energies, u_{0D2D3}, u_{0D3D4} and u_{0D4D5}, are found by integrating the shear stresses in the corresponding bar portions. The shear stresses and strains in the bar portions are analyzed by Equations (14) – (21).

Equations (14) and (15) that relate the angles of twist, φ_{D1} and φ_{D2}, with the shear strains at the surface of the bar portions are obtained by the integrals of Maxwell-Mohr.

$$\varphi_{D1} = \frac{\gamma_{D1D3}}{R_1}(l_1 + a) + \frac{\gamma_{D3D4}}{R_2}(l_2 - a) + \frac{\gamma_{D4D5}}{R_3} l_3 \tag{14}$$

$$\varphi_{D2} = \frac{\gamma_{D2D3}}{R_2} a + \frac{\gamma_{D3D4}}{R_2}(l_2 - a) + \frac{\gamma_{D4D5}}{R_3} l_3 \tag{15}$$

Equations (16) - (19) relate the shear strains, γ_{D1D3}, γ_{D2D3}, γ_{D3D4} and γ_{D4D5}, with time.

$$\gamma_{D1D3} = v_{D1D3} t \tag{16}$$

$$\gamma_{D2D3} = v_{D2D3} t \tag{17}$$

$$\gamma_{D3D4} = v_{D3D4} t \tag{18}$$

$$\gamma_{D4D5} = v_{D4D5} t \tag{19}$$

where v_{D1D3}, v_{D2D3}, v_{D3D4} and v_{D4D5} are parameters.

Equations (20) and (21) are composed by analyzing the equilibrium of sections, D_3 and D_4, of the bar.

$$\iint_{(A_1)} \tau_{D1D3} R dA = \iint_{(A_2)} \tau_{D3D4} R dA \tag{20}$$

$$\iint_{(A_2)} \tau_{D3D4} R dA = \iint_{(A_3)} \tau_{D4D5} R dA \tag{21}$$

where the shear stresses, τ_{D1D3}, τ_{D3D4}, τ_{D3D4} and τ_{D4D5}, are found by inserting the shear strains, γ_{D1D3}, γ_{D2D3}, γ_{D3D4} and γ_{D4D5}, in Equation (4).

Equations (14), (15), (20) and (21) are used to determine v_{D1D3}, v_{D2D3}, v_{D3D4} and v_{D4D5}.

The SERR determined by Equation (11) is checked-up by Equation (22):

$$G = \frac{dU^*}{dA_{cr}} \quad (22)$$

where

$$dA_{cr} = 2\pi R_1 da, \quad (23)$$

$$U^* = \iiint_{(V_{D1D2})} u^*_{0D1D3}\, dV + \iiint_{(V_{D2D3})} u^*_{0D2D3} dV + \iiint_{(V_{D3D4})} u^*_{0D3D4_0} dV + \iiint_{(V_{D4D5})} u^*_{0D4D5}\, dV \quad (24)$$

Equation (25) is applied for determining the specific complementary strain energy, u^*_{0D1D3}.

$$u^*_{0D1D3} = \tau_{D1D3}\gamma_{D1D3} - \int \tau_{D1D3} d\gamma \quad (25)$$

The other specific complementary strain energies, u^*_{0D2D3}, u^*_{0D3D4} and u^*_{0D4D5}, are derived by inserting the corresponding stresses and strains in Equation (25).

3. Results

The solution obtained is used in order to examine how the SERR behaves when some basic parameters of the problem considered are varied. The SERR behavior is illustrated by the graphs in Fig. 2, Fig. 3 and Fig. 4. The results in these figures are obtained at $l_1 = 0.400$ m, $l_2 = 0.500$ m, $l_3 = 0.400$ m, $R_1 = 0.005$ m, $R_2 = 0.007$ m, $R_3 = 0.009$ m, $v_{D2} = 5\times10^{-7}$ rad/s, $v_{D4} = 7\times10^{-7}$ rad/s, $b = 0.4$, $f = 0.4$, $g = 0.6$ and $h = 0.6$.

Figure 2 gives a notion for the behavior of the SERR when the parameter, v_{D1}, that controls φ_{D1} is varied. The SERR behavior is characterized by a fast growth with rise of the parameter in question (Fig. 2).

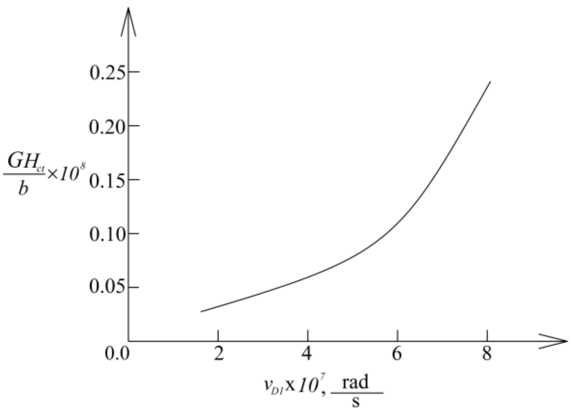

Figure 2. The SERR versus v_{D1}.

The reason for this is that higher v_{D1} means that the bar is loaded more heavily.

Figure 3 shows how the SERR changes when the value of P_{ct} is varied in the range between 0.5 and 2.0. It is apparent from Fig. 3 that increasing of P_{ct} increases the SERR.

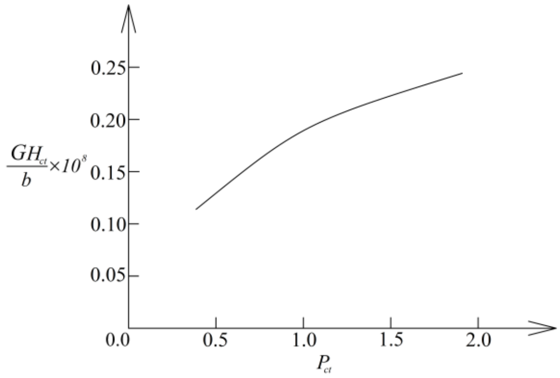

Figure 3. The SERR versus P_{ct}.

The graph in Fig. 4 exhibits how the SERR is affected by the parameter, $\dot{\gamma}_{oct}$.

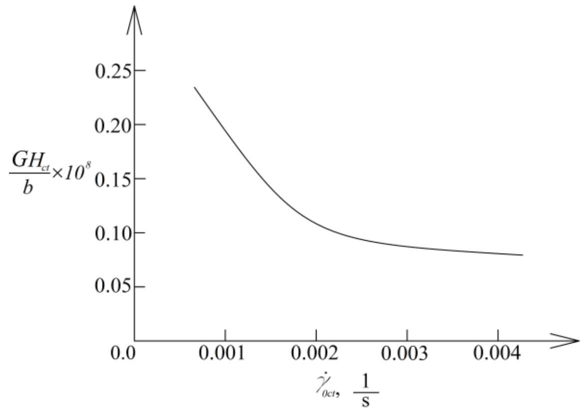

Figure 4. The SERR versus $\dot{\gamma}_0$.

The trend of decreasing of the SERR as $\dot{\gamma}_{oct}$ is increased is expected since higher $\dot{\gamma}_{oct}$ means lower shear stress at a particular shear strain in view of the material non-linearity.

4. Conclusion

The lengthwise fracture problem in a load-carrying inhomogeneous non-linear elastic bar under torsional loading is treated analytically. The non-linear elastic behavior of the bar depends on the twist velocity. This dependency is taken into account in the current analysis (this is one of the fundamental aims of the analysis). Thus, the SERR that is determined here is a function of the first derivative of the shear strains with respect to time. Results describing how the SERR behaves when some parameters of the solution obtained are varied are reported in graphical form. These results give reason for the conclusion that the SERR is strongly affected by the parameters, v_{D1}, P_{ct} and $\dot{\gamma}_{oct}$. The rise of the value of

v_{D1} has a negative influence of the lengthwise fracture behavior since the SERR grows. Similar is the effect of the parameter, P_{ct} . The results show a positive effect of the rise of $\dot{\gamma}_{oct}$ on the lengthwise fracture behavior because the SERR declines.

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