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THEORETICAL ANALYSIS OF DELAMINATION IN A VISCOELASTIC MULTILAYERED BAR BUILT-UP AT BOTH ENDS

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Abstract. *This paper reports the results of a theoretical consideration of the delamination problem in a multilayered load-bearing bar of rectangular cross-section loaded in time-dependent torsion. The bar is built-up at both ends. Besides, the bar is supported by a spring and a dashpot. The bar has two portions with different thickness. There is a delamination near the border between the two portions of the bar. The viscoelastic behavior of the bar is treated by a model that is subjected to shear stresses which vary with time. The torsion moments in the bar portions are determined by analyzing the time-dependent equilibrium with taking into account the effects of the spring and dashpot supports. Then these torsion moments are used to find-out the time-dependent strain energy in the bar. The strain energy release rate (SERR) for the delamination is obtained by differentiating the time-dependent strain energy with respect to the delamination area. The time-dependent compliance of the bar is analyzed to verify the SERR. Effects of the external loading, locations of the spring and dashpot supports, bar geometry, material properties and other parameters on the SERR are evaluated and discussed. The results of the analysis are presented in forms of various graphs illustrating the change of the SERR.*

Keywords: *Multilayered Bar, Delamination, Spring, Viscoelastic Behavior, Time-dependent Torsion*

1. INTRODUCTION

The multilayered materials are attracting much attention in the field of mechanical engineering and in the construction industry [1]-[7]. They are used nowadays as popular engineering materials due their high performance [8]-[12]. Useful analyses of multilayered structural materials like laminated composites are performed in [13] – [15]. Sandwich cylindrical and spherical shells made of laminated composite are studied numerically in [13]. The effect of porosity on the behavior of laminated composite shells is investigated in [14]. The behavior of laminated composite plates under different loading conditions is examined in [15]. However, multilayered materials are employed with caution especially in load-bearing applications because of their relatively bad delamination behavior [16]-[21]. In fact, delamination is one of the most serious reasons for the poor reliability of these materials in structural applications. Therefore, the development and improvement of multilayered materials require conducting of intensive research of the delamination problem.

This paper studies a multilayered viscoelastic bar with a delamination crack. The bar has two portions of different thickness. The delamination is located in the thicker portion of the bar. The bar is loaded by a time-dependent torsion moment. Solution of the SERR is derived. One of the novelties in the paper is that the solution derived accounts for the effect of two kinds of external resistance torsion moments acting on the bar. One of these moments is proportional to the angle of twist, while the other is proportional to the first derivative of the angle of twist with respect to time. External resistance torsion moments proportional to the angle of twist can appear in practice when the bar is supported by rotational springs located in planes perpendicular to the centroidal axis of the bar. The other kind of external resistance torsion moments, i.e. these proportional to the first derivative of the angle of twist with respect to time in practice can be generated by dashpots. The solution obtained is used to see how the SERR is affected by the external resistance torsion moments acting on the bar. The effect of the loading parameter (that controls the external torsion moment applied on the bar) on the SERR is also studied.

2. MODEL FORMULATION

The static model of the load-carrying bar that is studied here is depicted in Fig. 1.

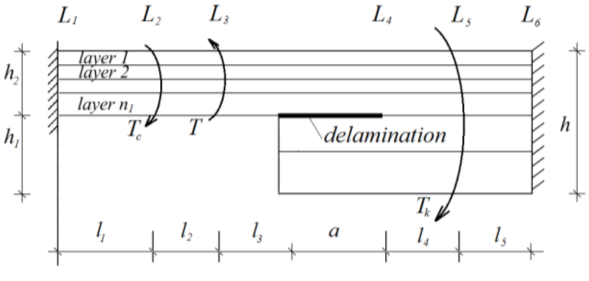

Figure 1. Static model of the bar.

The bar hosts a delamination of length, a, as depicted in Fig. 1. The bar is under torsion moment, T, applied in section, L_3. Equation (1) presents the change of T with time, t.

$$T = \delta t \tag{1}$$

where δ is the loading parameter.

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The bar viscoelastic behavior is modeled by the model depicted in Fig. 2. This model has two springs (with moduli of elasticity E_{Bi} and E_{Hi}) and two dashpots (with coefficients of viscosity η_{Qi} and η_{Di}) as depicted in Fig. 2.

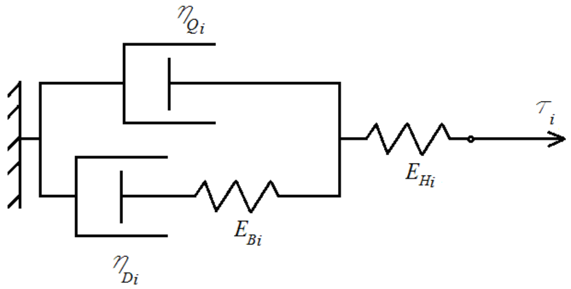

Figure 2. Schema of the viscoelastic model.

The model is applied in the current study since it has already been used in previous publications for treating the viscoelastic behavior of inhomogeneous load-bearing engineering structures [22]. It should also be noted that viscoelastic models representing combinations of springs and dashpots (similar to that shown in Fig. 2) are widely applied for describing the viscoelastic behavior of various engineering structures. Equation (2) presents the stress-strain-time relationship of the model that describes the viscoelastic behavior of the *i*-th layer of the bar [22].

$$\gamma_i = \frac{\tau_i}{\theta_i^2 t}\left(\frac{1}{\eta_{Qi}} - \frac{\beta_i}{\theta_i}\right)\left(e^{-\theta_i t} - 1\right) + \frac{\beta_i \tau_i}{2\theta_i} t + \frac{\tau_i}{\theta_i}\left(\frac{1}{\eta_{Qi}} - \frac{\beta_i}{\theta_i}\right) + \frac{\tau_i}{E_{Hi}} \tag{2}$$

where

$$\theta_i = \frac{E_{Bi}}{\eta_{Qi}}\left(1 + \frac{\eta_{Qi}}{\eta_{Di}}\right) \tag{3}$$

$$\beta_i = \frac{E_{Bi}}{\eta_{Di}\eta_{Qi}} \tag{4}$$

$$\tau_i = \delta_i t \tag{5}$$

$$i = 1, 2, ..., n . \tag{6}$$

Here τ_i and γ_i are the shear stress and the shear strain, n is the number of layers.

The bar is built-up at its both ends (Fig. 1). Thus, it can be written that

$$\varphi_{L1} = 0 \tag{7}$$

where φ_{L1} is the angle of twist of end, L_1, of the bar. Equation (8) expresses φ_{L1} as a function of torsion moments, T_1, T_2, T_3, T_4 and T_5, in the bar portions, L_1L_2, L_3L_4, L_4L_5 and L_5L_6.

$$\varphi_{L1} = \frac{T_1}{S} l_1 + \frac{T_2}{S} l_2 + \frac{T_3}{S}\left(l_3 + a\right) + \frac{T_4}{S_g} l_4 + \frac{T_5}{S_g} l_5 \tag{8}$$

where S and S_g are the rigidities in torsion of the bar at thickness, h_2 and h, respectively (S and S_g are determined by the approach from [23]). Equations (9) and (10) present the angles of twist of sections, L_2 and L_5, of the bar as functions of torsion moments.

$$\varphi_{L2} = \frac{T_2}{S} l_2 + \frac{T_3}{S}\left(l_3 + a\right) + \frac{T_4}{S_g} l_4 + \frac{T_5}{S_g} l_5 \tag{9}$$

$$\varphi_{L5} = \frac{T_5}{S_g} l_5 . \tag{10}$$

Equations (11), (12), (13) and (14) describe the equilibrium of sections, L_2, L_3, L_4 and L_5, of the bar.

$$T_1 + T_c + T_2 = 0 , \tag{11}$$

$$T_2 + T + T_3 = 0 , \tag{12}$$

$$T_3 + T_4 = 0 , \tag{13}$$

$$T_4 + T_k + T_5 = 0 , \tag{14}$$

where T_c and T_k are the external resistance torsion moments in sections, L_2 and L_5, of the bar (Fig. 1). Equations (15) and (16) determine T_c and T_k, respectively.

$$T_c = c\varphi_{L2} , \tag{15}$$

$$T_k = k\dot{\varphi}_{L5} , \tag{16}$$

where c and k are the constants, $\dot{\varphi}_{L5}$ is the derivative with respect to time.

Equations (7), (11), (12), (13) and (14) are used to determine T_1, T_2, T_3, T_4 and T_5.

After determining of T_1, T_2, T_3, T_4 and T_5, the SERR, G, is derived by Eq. (17).

$$G = \frac{dU}{dA} \tag{17}$$

where

$$dA = bda , \tag{18}$$

$$U = U_{L1L2} + U_{L2L3} + U_{L3L4} + U_{L4L5} + U_{L5L6} . \tag{19}$$

Here, b is the bar width, U is the strain energy in the bar.

Equations (20) – (24) are applied for determining the strain energies in the portions of the bar.

$$U_{L1L2} = l_1 \sum_{i=1}^{n_1} \iint_{(A_i)} u_{0L1L2i} dA_i \tag{20}$$

$$U_{L2L3} = l_2 \sum_{i=1}^{n_1} \iint_{(A_i)} u_{0L2L3i} dA_i \tag{21}$$

$$U_{L3L4} = (l_3 + a)\sum_{i=1}^{n_1} \iint_{(A_i)} u_{0L3L4i} dA_i \ , \qquad (22)$$

$$U_{L4L5} = l_4 \sum_{i=1}^{n} \iint_{(A_i)} u_{0L4L5i} dA_i \ , \qquad (23)$$

$$U_{L5L6} = l_5 \sum_{i=1}^{n} \iint_{(A_i)} u_{0L5L6i} dA_i \ , \qquad (24)$$

where

$$u_{0i} = \frac{1}{2}\tau_i \gamma_i \qquad (25)$$

is the specific strain energy, n_1 is the number of layers in the thin portion of the bar.

The SERR is checked-up by using the compliance method.

3. Analysis results

The results of the analysis of the SERR are reported in Fig. 3 and Fig. 4.

The data used in the SERR analysis are $l_1 = 0.150$ m, $l_2 = 0.150$ m, $l_3 = 0.100$ m, $a = 0.200$ m, $l_4 = 0.150$ m, $l_5 = 0.200$ m, $h = 0.012$ m, $b = 0.010$ m, $n_1 = 2$ and $n = 4$. The goal of the analysis is to see how the SERR is affected by the loading parameter and the constants, c and k, of the external resistance torsion moments acting on the load-bearing bar.

Typical results for the change of the SERR when δ and c vary are given in Fig. 3. It appears that the magnitude of theoretical effect of δ and c on the SERR is significant (Fig. 3). An interesting point is that the SERR comes down when the value of c rises. The SERR goes up when the δ value grows (Fig. 3).

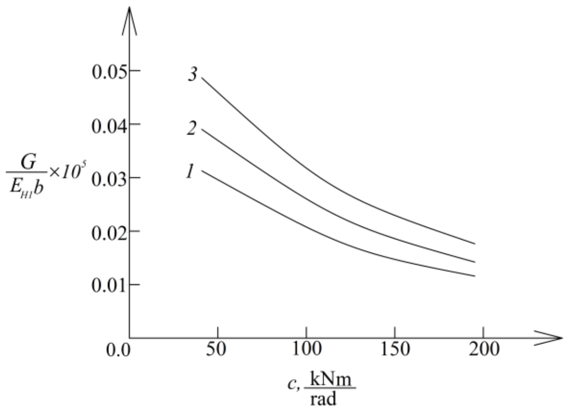

Figure 3. The SERR versus c (curve 1 – for $\delta = 10^{-6}$ kNm/s, curve 2 – for $\delta = 10^{-4}$ kNm/s and curve 3 – for $\delta = 10^{-2}$ kNm/s).

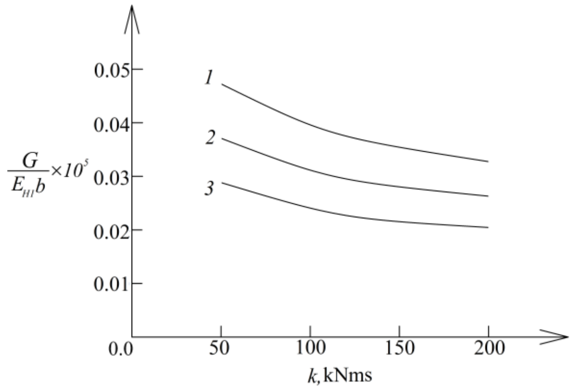

Figure 4. The SERR versus k (curve 1 – for $\frac{E_{H2}}{E_{H1}} = 0.5$, curve 2 – for $\frac{E_{H2}}{E_{H1}} = 1.0$ and curve 3 – for $\frac{E_{H2}}{E_{H1}} = 2.0$).

An analysis of the change of the SERR provoked by k and E_{H2}/E_{H1} ratio is also performed. It turned out that the results for the SERR are very sensitive for the considered range of k and E_{H2}/E_{H1} ratio (Fig. 4). Irrespective of E_{H2}/E_{H1} ratio, the SERR comes down with rise of the k value.

4. Conclusion

The delamination problem in multilayered viscoelastic load-bearing bars under time-dependent torsion is studied. The effects of external resistance torsion moments that are proportional to the angle of twist and to its first derivative with respect to time are considered. The main goal of the study is to see to what extent the SERR is affected by these resistance moments. In fact, studying the effects of these resistance torsion moments is the main novelty of the present paper (these effects are not analyzed in the previous works on delamination [17]-[21]). Based on the analysis performed, it can be stated that the presence of such moments affects the SERR in a high degree. Concerning the effect of the constants, c and k, it can be summarized that the SERR comes down when the values of c and k increase. The SERR comes down also when E_{H2}/E_{H1} ratio grows. It is demonstrated that rise of the loading parameter – as expected – induces a significant rise of the SERR. The analysis of the SERR presented in this paper can be applied in engineering practice when dealing with delamination problem in multilayered beams under torsion in the presence of external resistance torsion moments (for example, a reliable assessment of the effects of these moments on the SERR can be carried-out).

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