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ALLOWED AND FIRST FORBIDDEN β⁻- DECAY STUDY OF $^{16}N \longrightarrow ^{16}O$ IN RANDOM PHASE APPROXIMATION FRAMEWORK

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Abstract. *Within a developed and sophisticated particle-hole random phase approximation, a systematic study of the β^--decay of ^{16}N to ^{16}O has been carried out. The theoretical framework used starts from a mean-field calculation with a phenomenological Woods-Saxon potential that includes spin-orbit and coulomb terms to get single-particle energies and wave functions. A schematic residual surface delta interaction (SDI) is then introduced on top of the mean-field and is treated within a random phase approximation (RPA). The parameters of this residual force are optimized for each individual state to reproduce the experimental excitation energies. Then, beta-decay properties are calculated for the possible allowed transitions, as well as for the first forbidden unique transition. In this approach, the endpoint energy, comparative and partial half-lives of theoretically possible transitions are calculated. The final results for the optimized calculation are reasonable and close to the available experimental data.*

Key words: *β- decay, transition, residual interaction, single-particle*

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1. INTRODUCTION

Over the years, nuclear models are improved to study nuclear structure. The mean-field approximation (MFA) is frequently used in nuclear structure description. The validity of MFA has been limited to phenomenological mean-field potential [1]. A complete set of mean-field potential consists of Woods-Saxon (WS), coulomb and spin-orbit coupling. Therefore, in order to extract nuclear single-particle wave functions and relevant energy spectrum, the radial A-nucleon Schrodinger equation for a complete mean-field Hamiltonian should be numerically solved.

The remaining two body interactions so called "residual interaction" are considered calculating as a perturbed part. As a result of this part, the final configurations of nucleons in nucleus are mixed [2]. The mixed configurations of states one-particle one-hole nuclei like ^{16}O and ^{16}N isotopes are characterized in either Tamm-Dancoff Approximation (TDA) and proton-neutron Tamm-Dancoff Approximation (pnTDA) [3], or Random-Phase Approximation (RPA) and proton-neutron Random-Phase Approximation (pnRPA) methods. The RPA & pnRPA yields a description of collective nuclear excitation in terms of an eigenvalue problem involving a non-Hermitian matrix [4].

The surface delta interaction (SDI) is considered to be a suitable residual interactionthat reproduces the qualitative behavior of nucleon-nucleon scattering data [2]. SDI parameters should be determined in order to construct a reasonable agreement between theoretical energy spectrum and the measured nuclear level energies, which is an optimization math problem. There are several Artificial intelligence (AI) algorithms to solve similar problems. In this approach, the genetic algorithm is employed to converge theoretically calculated energies with the experimental energy spectrum.

The study of nuclear decay rates via β-decays is an excellent test of the validity of nuclear model for estimating nuclear states and wave functions [1].

In this research, the β-decay transition quantity of ^{16}N to ^{16}O has been calculated via the RPA and TDA methods.

2. METHOD

2.1. Energy eigenvalues problem of one-particle-one-hole ^{16}N and ^{16}O nuclei in RPA & pnRPA approximation

In this section, the single-particle energies and corresponding wave functions of ^{16}O and ^{16}N isotopes are numerically obtained in our solution of mean-field Schrodinger equation, with Woods-Saxon plus coulomb and spin-orbit mean-field potentials. This single-particle Schrodinger equation is solved by the QR factorization method [5]. These single-particle bases are shown in Figs. 1 and 2 and the corresponding single-particle energies are presented in Fig. 3.

Then, the SDI residual interaction is used in RPA and pnRPA approach and the corresponding mixed configurations of Eigen functions are calculated in order for charge-conserving excited states of ^{16}O and charge-changing ground and excited states of ^{16}N

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isotopes. The SDI residual interaction is defined as [2, 3]:

$$V(ij) = -4\pi A_T \delta(\Omega_{ij}) \quad (1)$$

Where, the isospin T=0(or 1) and the constants A_0 and A_1 are SDI parameters. The SDI parameters can be a certain amount for all states. In this study, the SDI parameters are evaluated by utilizing the genetic algorithm to achieve maximum consistency between theoretical and corresponding experimental spectrum by minimizing f_{RMS} value for each unique J^{π}. We defined the f_{RMS} as [3]:

$$f_{RMS} = \sqrt{\sum_n f_n} \quad (2)$$

where,

$$f_n = \min\left\{(E_m^{(th)} - E_n^{(\exp)})^2 \mid m \in W,\ n \le m \le (n+1)(1-\delta_{n0})\right\}$$

Therefore, the energy states andcorresponding wave functions are calculated with RPA & pnRPA states of ^{16}O and ^{16}N. Some of the evaluated energy states of ^{16}N &^{16}O, which are needed in the next calculations, are shown in Figs. 4 and 5, respectively.

2.2. Allowed and first-forbidden unique β– decay of $^{16}N \rightarrow ^{16}O$

The main objective of this research is to analyze theoretically β transitions between the states of ^{16}N and ^{16}O, as it is shown in Fig. 6 [6].

According to the Fig. 6, and taking into account β-transition theory [1, 2], transitions to 3_1^-, 1_1^-, $2_1^-$and 1_2^- states are of the Gamow-Teller types. Fermi transition type is only possible to the 2^- final state. The 2^- ground state transition of ^{16}N nuclei to the 0^+ ground state of ^{16}O nuclei is considered as a first-forbidden unique beta-decay. The end point energies of transitions can be obtained by applying the energy states shown in figs. 4 & 6 into:

$$E_0 = 1 + (Q_\beta - E_{ex})/(M_e c^2) \quad (3)$$

whose results are presented in Table 1.

3. Results and discussion

By applying the computed wave functions, the calculated reduced transition probabilities are used in the comparative half-lives indications. The logarithms of the comparative half-lives are presented in Table 2.

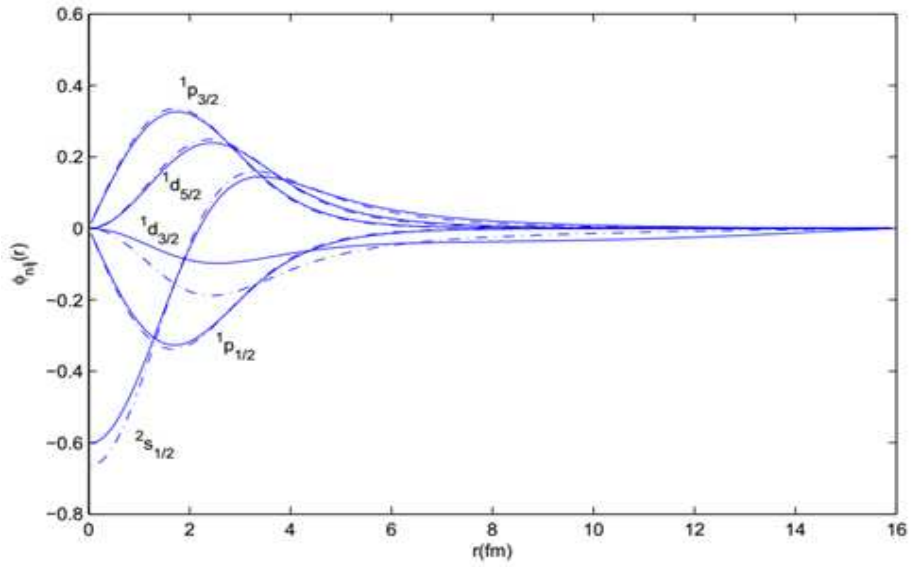

Figure 1. The plots of neutrons and protons single particle wave functions in order with dashed point lines and solid lines, with reference to Woods-Saxon, coulomb and spin orbit interactions of ^{16}O isotope.

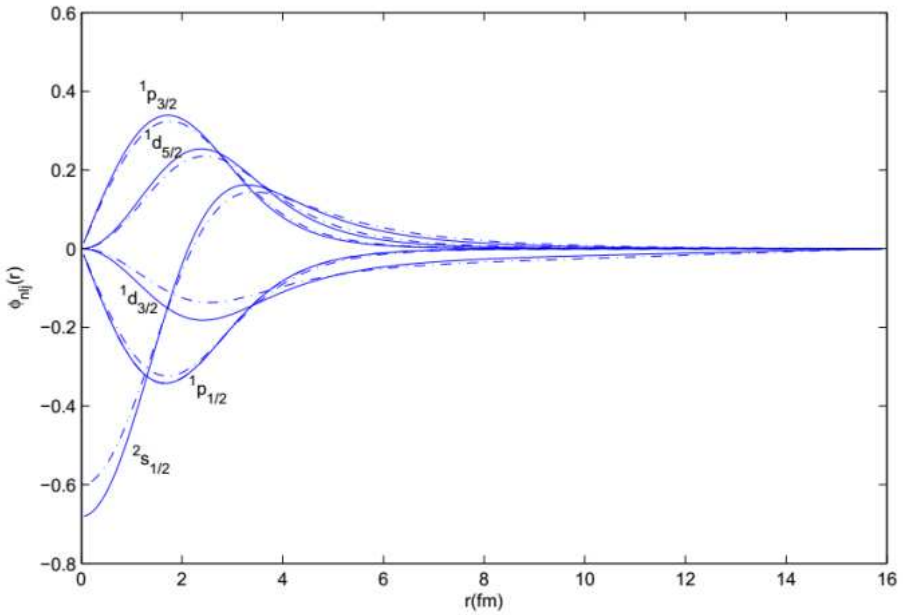

Figure 2. The plots of neutrons and protons single particle wave functions in order with dashed point lines and solid lines, with reference to Woods-Saxon, coulomb and spin-orbit interactions of ^{16}N nucleus

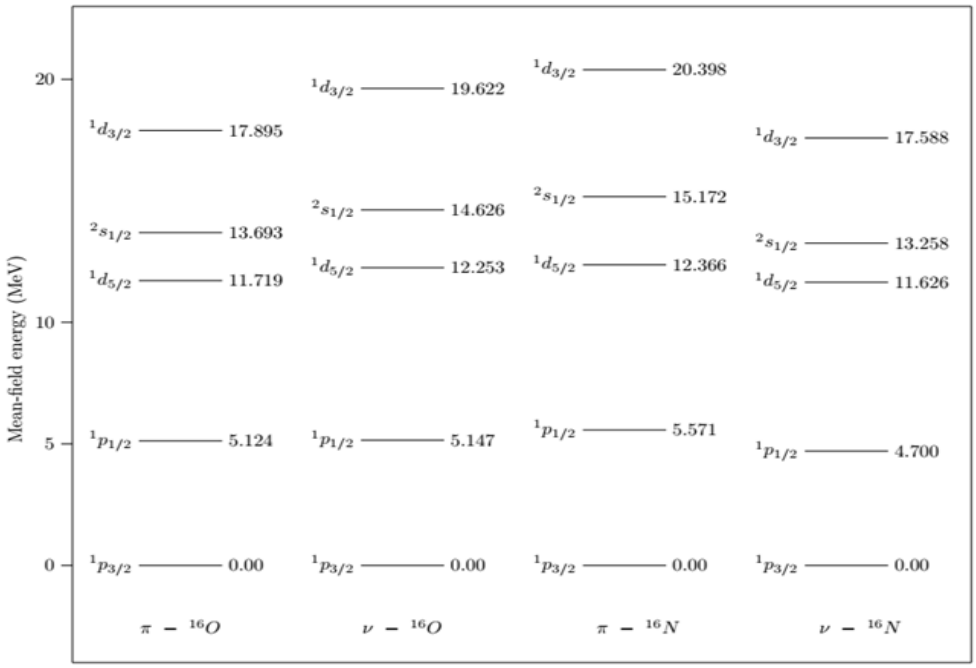

Figure 3. Proton(π) and neutron(ν) single-particle energy eigenvalues evaluated from QR factorization solution of MF Schrodinger equation of the particle-hole valence spaces which are used to calculate spectra of the^{16}O and ^{16}N nuclei where are shown in Figures 4 and 5

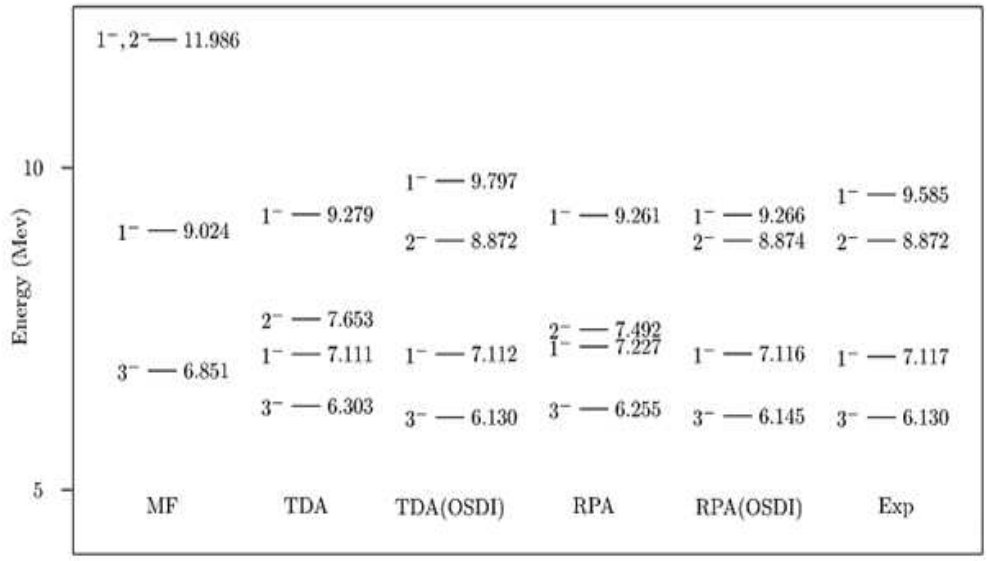

Figure 4. Theoretical energy levels of ^{16}O in pure MF configuration and TDA+SDI [3], RPA+SDI, and TDA, RPA with optimized-SDI parameters (TDA(OSDI) [3]), (RPA(OSDI)) in 1p–1 –(1d –2s) valance space along with the experimental levels [6]

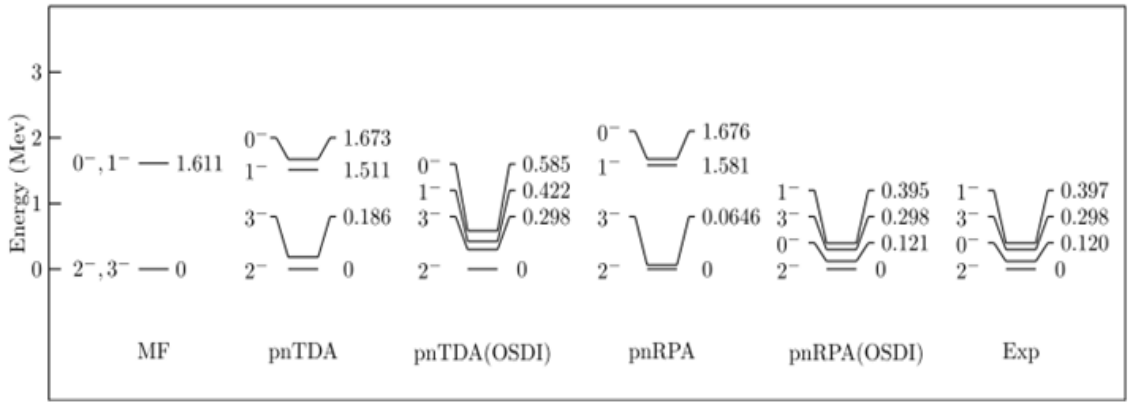

Figure 5. Theoretical energy levels of ^{16}N in pure MF configuration and pnTDA+SDI [3], pnRPA+SDI and pnTDA, pnRPA with optimized-SDI parameters (pnTDA(OSDI) [3]), (pnRPA(OSDI)) in 1p–1–(1d – 2s) valance space along with the experimental levels [6]

Theoretical partial decay half-lives are calculated using the values presented in Tables 1&2 and the total decay half-life is then given by:

$$\frac{1}{t_{1/2}} = \sum_k \frac{1}{t_{1/2}^{(k)}} \qquad (4)$$

A summary of the results of such calculations and comparisons with partial and total experimental half-lives of β-decay transitions of ^{16}N isotope are presented in Table 3.

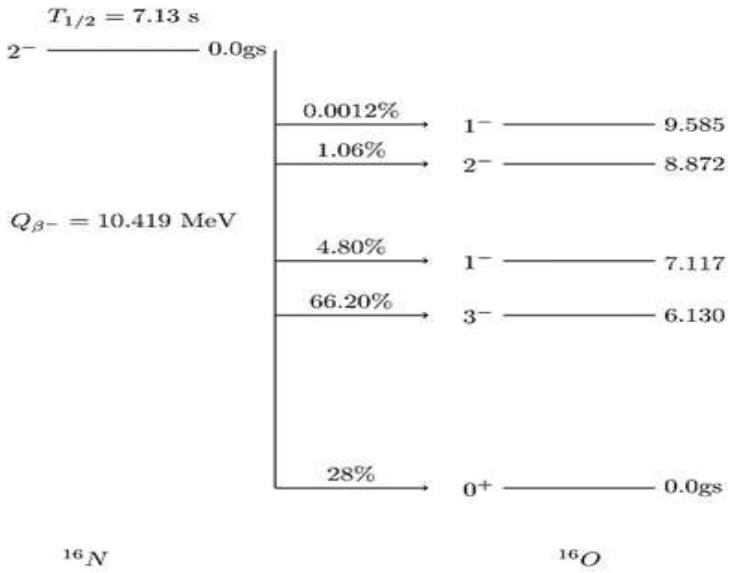

Figure 6. Shows diagram of β-decay transitions between the ground state of ^{16}N and the ground and excited states of ^{16}O. The experimental total half-life, decay Q-values, branching ratios, log *ft* values and excitation energies (in MeV) of the four first minus parity states of ^{16}O [6]

Table 1. The endpoint energies of ^{16}N→^{16}O transition for three theoretically calculated initial and final sets of Figures 4 & 5. These dimensionless quantities used to calculate the comparative half-lives which are presented in Table 2.

I → F	*TDA*[3]	Optimized *TDA*[3]	*RPA*	Optimized *RPA*	EXP[6]
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 3^-_1(^{16}\mathrm{O})$	9.054	9.393	9.147	9.394	9.393
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 1^-_1(^{16}\mathrm{O})$	7.474	7.471	7.245	7.462	7.462
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 2^-_1(^{16}\mathrm{O})$	7.452	4.027	6.727	4.023	4.027
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 1^-_2(^{16}\mathrm{O})$	3.231	2.216	3.265	3.257	2.632

Table 2. The logarithm of β-decay comparative half-lives (log ft) of one-particle one-hole transition of ^{16}N →^{16}O transitions

I → F	*TDA*[3]	Optimized *TDA*[3]	*RPA*	Optimized *RPA*	EXP[6]
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 0^+_{gs}(^{16}\mathrm{O})$	8.760	8.877	8.858	9.150	9.071
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 3^-_1(^{16}\mathrm{O})$	4.291	4.367	4.190	4.487	4.482
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 1^-_1(^{16}\mathrm{O})$	6.286	4.642	5.612	5.085	5.110
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 2^-_1(^{16}\mathrm{O})$	3.692	4.142	4.674	4.486	4.320
$2^-_{gs}(^{16}\mathrm{N}) \rightarrow 1^-_2(^{16}\mathrm{O})$	6.027	5.901	6.453	6.391	6.20

Table 3. Theoretical beta minus decay half-lives, $t_{1/2}$(sec) of $^{16}N \rightarrow ^{16}O$ transitions

I → F	*TDA*[3]	Optimized *TDA*[3]	*RPA*	Optimized *RPA*	EXP[6]
$2^-_{gs}(^{16}N) \rightarrow 0^+_{gs}(^{16}O)$	12.401	16.254	15.550	30.470	25.464
$2^-_{gs}(^{16}N) \rightarrow 3^-_1(^{16}O)$	8.150	8.081	6.142	10.651	10.770
$2^-_{gs}(^{16}N) \rightarrow 1^-_1(^{16}O)$	2121	48.229	525.968	134.627	148.542
$2^-_{gs}(^{16}N) \rightarrow 2^-_1(^{16}O)$	5.481	365.758	88.323	812.657	672.642
$2^-_{gs}(^{16}N) \rightarrow 1^-_2(^{16}O)$	9.218×10^4	6.326×10^5	2.319×10^5	2.037×10^5	5.942×10^5
Total Half-life	2.589	4.791	4.161	7.387	7.13

5. Conclusion

In this investigation, the Random Phase Approximation (RPA) and also its developed version, proton-neutron Random Phase Approximation (pnRPA) have been used to study of the one-particle one-hole ^{16}O and ^{16}N nuclei states in the full (1d-2s) - (1p–1) shell. The Woods-Saxon plus coulomb and spin-orbit interactions are adopted to calculate single-particle wave functions of Figs. 1 & 2, and the corresponding energies were presented in Fig. 3. Moreover, the SDI residual interaction was used in production of more realistic nuclear wave function and energy states of the aforementioned nuclei in (pn)RPA picture. The energy states were optimized with experiments in two pictures.

The achieved wave functions can be used in calculations of theoretical endpoint energy, log ft values, partial and total half-lives of β-decay branches from the ground state of ^{16}N to the 0^+,3^-, 1^- and 2^- states of the daughter ^{16}O nucleus. All of the calculated values are in accordance with those of experimental data. Furthermore, the application of the optimization on the particle-hole valance space acts coherently to increase the collectivity specially in the (pn)RPA picture. According to the obtained results, the use of optimized RPA approach is an appropriate method to deduce the experimental data.

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